SIMULTANEOUS 4-STOKES PARAMETER DETERMINATION USING A SINGLE DIGITAL IMAGE

GOVERNMENT USE

1. The invention described here may be made, used and licensed by the or for the U.S. Government for governmental purposes without paying us any royalty.

BACKGROUND

2. A seminal method to determine the state of polarization of a light beam using measurable quantities is the Stokes method, which involves four independent intensity measurements of the light beam. Each measurement corresponds to the intensity of the beam after it passes through each of four different filter system arrangements. The four Stokes parameters, sometimes called S_0, S_1, S_2 and S_3, are derived from these four measured intensities and form a four-element column vector in four-dimensional mathematical space.

3. Since the discovery of the Stokes method in 1852, many filter systems based thereon have been presented. Extracting polarization information from images is not new either. However, four separate images are used to calculate the Stokes parameters for each element in a scene. To date, a major problem still exists in using the Stokes method for acquiring polarization information from images. The problem occurs because it takes time to capture separate images. In the time it takes to acquire each image, the intensity or polarization state of points in the scene may change. This time factor would affect

polarization measurements taken outdoors where changing sun position or cloud conditions would change the intensity or polarization state of the light entering the filter system. In the laboratory, temperature, pressure, density or concentration variations associated with scene elements may change the polarization state of the light entering the filter system during the time required to record four separate images.

PRIOR ART

4. A relevant item of prior art is a patent to G.R. Gerhart and R.M. Matchko, "Method of Determining Polarization Profiles for Polychromatic Sources." US Patent # 5,734,473, issued March 31, 1998.

SUMMARY

5. Our method and apparatus for determining and displaying polarization profiles of a scene from a single digital image employs a four-system filter-imaging array. Each of the four systems attenuates the intensity of the light transmitted through it and creates an image of the scene. The four systems operate done simultaneously in real time. Three of the four systems consist of a linear polarizer positioned in front of an imaging lens. The other system consists of a retarder and a linear polarizer positioned in front of an imaging lens. The relative positions of the transmission axes of the linear polarizers and the fast axis of the retarder determine the attenuation of the intensity of the light transmitted through each of the four systems. A CCD (Charged Coupled Device) video camera, fitted with a narrow band color filter and camera lens, simultaneously captures

and records the four images produced by the four-system filter-imaging system. Each CCD video frame consists of four attenuated images of the scene. A computer program crops and registers selected corresponding elements from each scene-image. Each of the four-cropped images consists of a rectangular array of pixel values (a matrix) corresponding to the attenuated intensities of the light transmitted through each filter. A calibration equation converts pixel values in each of the four matrices to optical densities and then to relative intensities. The Stokes parameters are calculated for each pixel in the scene. Polarization parameters such as the degree of polarization, polarization azimuth angle and polarization ellipticity angle can be calculated for each pixel from the Stokes parameters associated with each pixel value. A unique pseudo-color scheme that utilizes the Poincarè sphere is used for encoding and displaying polarization parameters in the scene. The method associates RGB values with the normalized values of the Stokes parameters. Our apparatus, method and polarization-encoding scheme allows one to create video images of changing polarization parameters in real time.

BRIEF DESCRIPTION OF THE DRAWINGS

6. Figure **1** is a diagram of light rays traveling from a scene through an optical array having four optical systems, each system creating an independent image operating according to our method. Figure **2** is a diagram showing light from the scene being attenuated by the first system, which is comprised of a linear polarizer with a horizontal transmission axis. It also shows the image formation of the first system. Figure **3** is a diagram showing light from the scene attenuated by the second system, which is

comprised of a linear polarizer with a vertical transmission axis. It also shows the image formation of the second system. Figure **4** is a diagram showing light from the scene attenuated by the fourth system, which is comprised of a retarder whose fast axes is horizontal and by a linear polarizer with a transmission axis at an angle $\theta = 45$ degrees relative to the horizontal direction. It also shows the image formation of the fourth system. Figure **5** is a diagram showing light from the scene attenuated by the third system, which is comprised of a linear polarizer with a transmission axis oriented at an angle $\theta = 45$ degrees relative to the horizontal direction. It also shows the image formation of the third system. Figure **6** is a diagram of the Poincarè sphere. The center of the sphere is the origin of a rectangular Cartesian coordinate system. The sphere has a unit radius. Every polarization state is associated with a unique point in or on the sphere. The normalized Stokes parameters are represented by the x, y and z coordinates of a point on or inside of the sphere. Points inside the sphere correspond to partially polarized light, points on the surface of the sphere correspond to light that is completely polarized.

7. It will be noted that Figures 7 through 13 depict various views of our pseudo-color version of a Poincarè sphere. These figures, due to US Patent Office regulations, must be in black and white. However, it will be understood that our Poincarè sphere contains the colors red, blue, green and blends of these colors. In Figures 7 through 13, zones of our version of the Poincarè sphere that are primarily red, blue or green are designated by reference letters R, G and B , respectively. In particular, Figure **7** shows a pseudo-color visualization of the surface of the Poincarè sphere when viewing the sphere along the +z axis. Figure **8** shows a pseudo-color visualization of the surface of the Poincarè sphere

when viewing the sphere along the -z axis. Figure **9** shows a pseudo-color visualization of the surface and partial interior of the Poincarè sphere when viewing the sphere along the +x axis. Figure **10** shows the relationship between the polarization forms, the corresponding pseudo-colors and the corresponding location on the Poincarè sphere when viewing the sphere along the +x axis. Figure **11** shows a pseudo-color view of the surface and partial interior of the Poincarè sphere when viewing the sphere along the -x axis. Figure **12** shows the relationship between the polarization forms, the corresponding pseudo-colors and the corresponding location on the Poincarè sphere when viewing the sphere along the -x axis. Figure **13** shows the pseudo-coloring of the horizontal, equatorial plane of the Poincarè sphere. The colors along the perimeter of this plane are used to encode azimuth and ellipticity polarization angles in a scene.

DETAILED DESCRIPTION

8. Figure 1 shows elements of one acceptable design for an array 3 that simultaneously creates four different attenuated images 6, 9, 12 and 16 of scene 1. The component systems of optical elements in array 3 that attenuate light from scene 1 and create images 6, 9, 12 and 16 of scene 1 are shown individually in Figures 2 through 5. Some of the light from scene 1 passes through planes x_1-y_1, x_2-y_2, x_3-y_3 and x_4-y_4 in Figures 2, 3, 5 and 4, respectively. In these figures the variously subscripted x and y axes all lie in the same plane, the x-axes are all parallel to each other and the y-axes are all parallel to each other.

9.. Figure 2 shows light that passes through the x_1-y_1 plane, such as ray 20, and is transmitted through and attenuated by linear polarizer 4 that has its transmission axis 21 oriented at an angle θ with respect to the x_1-axis and the x_1-z_1 plane, θ being 0° in Figure 2, such that axis 21 lies along axis x_1'. The exiting attenuated light, such as ray 22, is then transmitted through imaging lens 5, which forms attenuated image 6 of scene 1. Figure 3 shows light that passes through the x_2-y_2 plane, such as ray 24, and is transmitted through and attenuated by linear polarizer 7 that has its transmission axis 25 oriented at an angle θ with respect to the x_2-axis and the x_2-z_2 plane, θ being 90° in Figure 3, such that axis 25 lies along axis y_2'. The exiting attenuated light, such as ray 26, is then transmitted through imaging lens 8, which forms attenuated image 9 of scene 1.

10. Figure 5 shows light that passes through the x_3-y_3 plane, such as ray 28, and is transmitted through and attenuated by linear polarizer 10 that has its transmission axis 29 oriented at an angle θ with respect to the x_3-axis and the x_3-z_3 plane, θ being 45° in Figure 5, where axis x_3' is parallel to axis x_3. The exiting attenuated light, such as ray 30, is then transmitted through imaging lens 11, which forms attenuated image 12 of scene 1.

11. Figure 4 shows light that passes through the x_4-y_4 plane, such as ray 32, and is transmitted through and attenuated by retarder 13 that has its fast axis 33 oriented at an angle Ω with respect to the x_4-axis and the x_4-z_4 plane, Ω being 0° in Figure 4, such that axis 33 lies along axis x_4'. Retarder 13 causes a phase difference ε between components of any given light wave passing through the system, ε having a different value for

different wavelengths. The retarder may be of any anisotropic material. Specifically, the following relationship exists for a quarter-wave plate made of quartz:

$$\varepsilon = \frac{\pi}{2}\left(\frac{\lambda_T - 50.876}{\lambda - 50.876}\right) \tag{1}$$

where λ is any visible wavelength and λ_T is that wavelength which produces $\varepsilon = \pi/2$, sometimes called the tuned wavelength. This relation is further discussed in US Patent 5,734,473 noted above. The exiting attenuated light, such as ray 34, is then transmitted through and attenuated by linear polarizer 14 that has its transmission axis 35 oriented at an angle θ with respect to the x_4-axis and the x_4-z_4 plane, θ being 45° in Figure 4, where axis x_4'' is parallel to axis x_4. The exiting attenuated light, such as ray 36, is then transmitted through imaging lens 15, which forms attenuated image 16 of scene 1.

12. Light from images 6, 9, 12 and 16, such as rays 23, 27, 31 and 37 (Figures 2, 3, 5 and 4, respectively) are transmitted through a color filter 17 (Figure 1), which selects a given bandwidth, the average of which becomes λ in equation (1) above. The exiting light from color filter 17 is transmitted through a camera lens 18, which forms a collective image 38 of the scene images 6, 9, 12 and 16 on the CCD array 19.

13. Image **38** is downloaded into a computer and a computer program crops selected corresponding elements from each of the four scene images. Scene image **6** is cropped to form image **39**, scene image **9** is cropped to form image **40**, scene image **12** is cropped to

form image **41** and scene image **16** is cropped to form image **42**. The pixel values of image **39** form the matrix M_1, the pixel values of image **40** form the matrix M_2, the pixel values of image **41** form the matrix M_3 and the pixel values of image **41** form the matrix M_4.

14. Since the Stokes parameters require intensity (I) measurements and the CCD array records RGB (red, blue and green) pixel values (X), a relationship between X and I must be obtained for the CCD array. One calibration method of obtaining this relationship is to pass an incident beam of collimated light of known intensity through neutral density filters of different known optical densities (Y) and record the average X for each Y. Alternatively, instead of using an incident beam of known intensity, one may measure the intensity of the beam exiting the neutral density filter. Curve-fitting yields Y as a function of X,

$$Y = f(X). \tag{2}$$

Since some CCD detectors are multi-channel arrays, a relationship between X and Y must be obtained for each channel.

Optical density is related to intensity through the equation

$$I = 10^{-Y} \tag{3}$$

Substituting equation (2) into equation (3) yields the CCD calibration equation

$$I = 10^{-f(X)} \tag{4}$$

Using equation (4), each pixel value, X, in each of the matrices M_1, M_2, M_3 and M_4 can be converted to an intensity value producing the new matrices I_1, I_2, I_3 and I_4 respectively.

15. The four Stokes parameters, S_0, S_1, S_2 and S_3, are then derived from the elements of the four intensity matrices I_1, I_2, I_3 and I_4 as follows:

$$\begin{aligned} S_0 &= I_1 + I_2 \\ S_1 &= I_1 - I_2 \\ S_2 &= 2\,I_3 - S_0 \\ S_3 &= \frac{2\,I_4 - S_0 - S_2 \cos\varepsilon}{\sin\varepsilon} \end{aligned} \tag{5}$$

Each of the elements in the matrices S_0, S_1, S_2 and S_3 correspond to a particular point in scene 1. For example, corresponding elements $s^{(0)}_{11}$, $s^{(1)}_{11}$, $s^{(2)}_{11}$ and $s^{(3)}_{11}$ from the four Stokes parameter matrices are associated with a point (x,y) in scene 1. Therefore, the polarization state of any point (x,y) in scene 1 can be determined from

$$\sin 2\chi = \frac{S_3}{\sqrt{S_1^2 + S_2^2 + S_3^2}} \qquad \tan 2\psi = \frac{S_2}{S_1} \qquad P = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \tag{6}$$

where χ is the polarization ellipticity angle, ψ is the polarization azimuth angle and P is the degree of polarization.

16. In addition to the above technique, we have invented a unique scheme that utilizes the Poincarè sphere (Figure 6) for encoding and displaying polarization parameters in a scene. In this scheme, the normalized values of the Stokes parameters are obtained by dividing S_0, S_1, S_2 and S_3 by S_0. As shown in Figure 6, the parameters S_1, S_2 and S_3 correspond to the x, y and z coordinates of points inside or on of the surface of the sphere

respectively. Points inside the sphere correspond to partially polarized light ($0 < P < 1$), whereas points on the surface of the sphere correspond to light that is totally polarized with $P = 1$.

17. The Stokes parameters are then encoded in a scene by assigning RGB (red, blue and green) values to the normalized values of S_1, S_2 and S_3 at each pixel site in the scene as follows:

$$R = \text{int}[127.5\ (1 - S_1)],\ G = \text{int}[127.5\ (1 - S_2)] \text{ and } B = \text{int}[127.5\ (1 - S_3)] \tag{7}$$

Where "int" is the integer function. Converting each pixel of a scene in accordance with equation 7 will result in a color map of the scene which characterizes the polarization of any selected area therein. For example, a given area *A* of the scene may have unpolarized light, where $S_1 = S_2 = S_3 = 0$. Unpolarized light corresponds to middle gray ($R = G = B = 127$) at the center of the Poincarè sphere, and thus area *A* will be a middle grey color on the aforementioned color map. Likewise, in general, any unpolarized or weakly polarized light is middle gray or unsaturated in the primary colors.

18. A method of encoding only P, the degree of polarization, is to covert each pixel of a scene into a corresponding 8-bit digital representation by the equation

$$\text{pixel value} = 255\ P$$

Encoding the pixels in this manner will produce a monochrome or grey-scale image, wherein the black areas correspond to light that has zero polarization, the white areas correspond to light that is 100 percent polarized, and areas of varying shades of grey correspond to light having varying degrees of polarization.

19. Still other options in our scheme assign RGB values to the azimuth polarization angle or assign RGB values to the ellipticity polarization angle. Both of these angles are essential parameters when desiring to represent a complete polarization profile. A method of displaying either one these angles for each pixel in a scene is to assign a different color to each specific size of that angle. The polarization azimuth angle, ψ, assumes values from 0 to 180 degrees while the ellipticity angle, χ, varies from -45 to 45 degrees. The ellipticity angle is positive for right-handed polarization and negative for left-handed polarization. Figure **13** shows the pseudo-coloring of the horizontal, equatorial plane of the Poincarè sphere. We use the colors along the perimeter of this plane to encode azimuth and ellipticity polarization angles in a scene. Each color along the perimeter of this cross-section of the Poincarè sphere corresponds to a unique combination of a ψ-value and a χ-value.

20. The spherical polar coordinates for any point on or inside the Poincarè sphere is given by

$$S_1 = P \cos 2\chi \cos 2\xi \qquad S_2 = P \sin 2\chi \cos 2\xi \qquad S_3 = P \sin 2\xi \tag{9}$$

where $x = S_1$, $y = S_2$, $z = S_3$, P (the degree of polarization) is the radius of the sphere and the origin of a Cartesian coordinate system is at the center of the sphere. For points along the perimeter of the equatorial plane of the Poincarè sphere $P = 1$ and $\chi = 0$. Using $P = 1$ and $\chi = 0$ and substituting equation (9) into equation (7) yields

$$R = \mathrm{int}[127.5\,(1 - \cos 2\psi)],\ G = \mathrm{int}[127.5\,(1 - \sin 2\psi)] \text{ and } B = 127 \tag{10}$$

Equation (10) contains the RGB values used to encode the ψ-values into a scene.

Substituting χ for ψ in equation (10) produces

$$R = \mathrm{int}[127.5\,(1 - \cos 2\chi)],\ G = \mathrm{int}[127.5\,(1 - \sin 2\chi)] \text{ and } B = 127 \tag{11}$$

Equation (11) contains the RGB values used to encode the χ-values into a scene.

21. We do not desire to be limited to the exact details of construction or method shown herein since obvious modifications will occur to those skilled in the relevant arts without departing from the spirit and scope of the following claims.